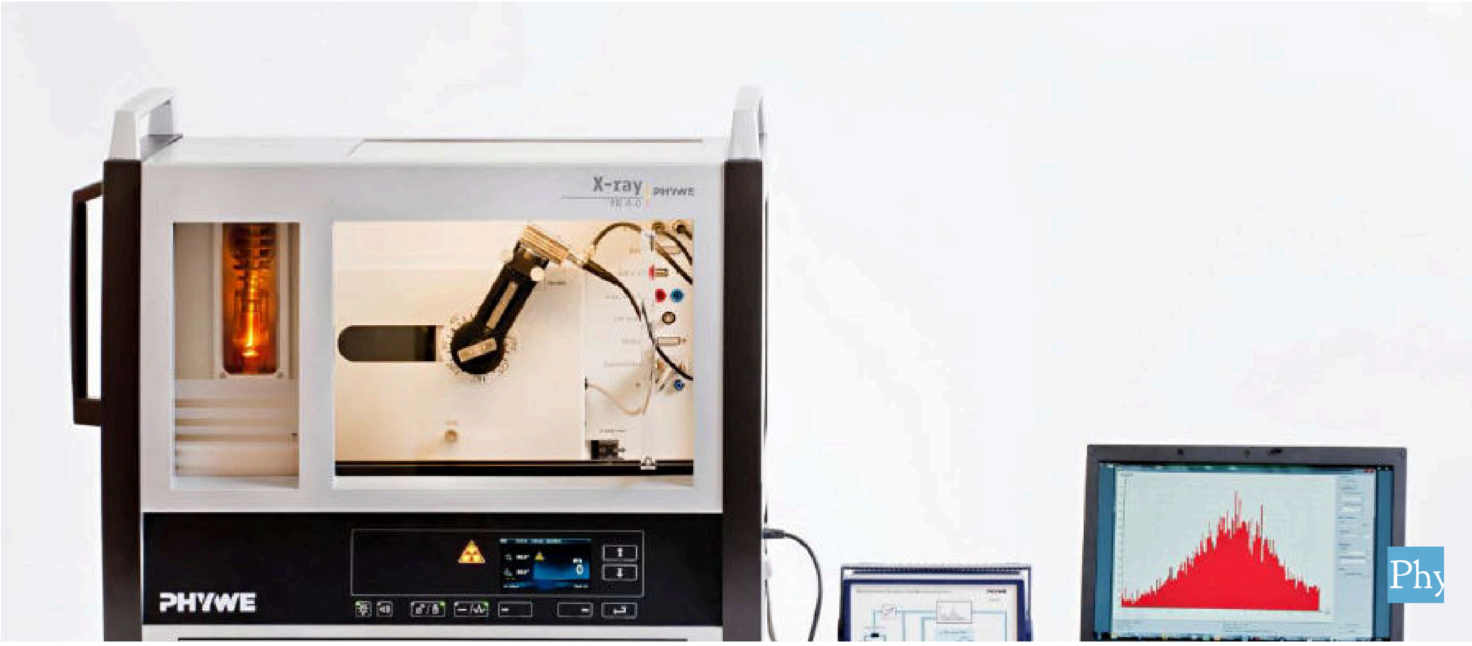
Compton effect - energy-dispersive direct measurement



Physics > Modern Physics > Production & use of X-rays

Difficulty level	Group size	Preparation time	Execution time
hard	2	45+ minutes	45+ minutes

PHYWE excellence in science

General information

Application

PHYWE excellence in science

Setup

Most applications of X rays are based on their ability to pass through matter. Since this ability is dependent on the density of the matter, imaging of the interior of objects and even peaple becomes possible. This has wide usage in fields such as medicine or security.

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Other information (1/2)

Prior knowledge

The prior knowledge for this experiment is found in the Theory section.

Main principle

Photons of the molybdenum K_α X-ray line are scattered at the quasi-free electrons of an acrylic glass cuboid. The energy of the scattered photons is determined in an angle-dependent manner with the aid of a swivelling semiconductor detector and a multi-channel analyser.

Other information (2/2)

Learning objective

The goal of this experiment is to get to investigate the compton effect.

Tasks

1. Calibrate the semiconductor energy detector.
2. Energy determination of the photons of the W-L_α-line that are scattered through an acrylic glass element as a function of the scattering angle.
3. Compare the measured energy values of the lines of scatter with the calculated energy values.
4. Calculate the Compton wavelength of electrons and a comparison of this value with the corresponding value of the 90° scattering.

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Theory (1/2)

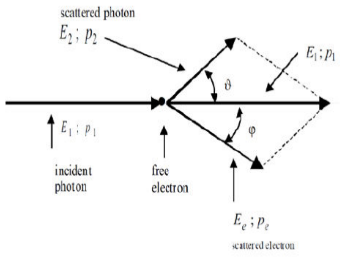


Fig. 1: Momentum and energy relation of the Compton effect

Figure 1 shows a schematic representation of the Compton Effect.

Due to the interaction with a free electron in the solid element, the incident photon loses energy and is deflected from its original direction with the scattering angle θ. The previously resting electron absorbs additional kinetic energy and leaves the collision point under the angle ϕ.

$$E_2 = \frac{E_1}{1 + \frac{E_1}{m_e c^2}(1 - \cos(\theta))} \qquad (1)$$

Photon energy before and after the collision	E_1 resp. E_2
Equivalent	1 eV = $1.6021 \cdot 10^{-19}$ J
Scattering angle	θ

Speed of light in vacuum

c = $2.988 \cdot 10^{8}$ m/s

Rest mass of the electron

m_e = $9.109 \cdot 10^{-31}$ kg

Theory (2/2)

After the collision, the photon has a lower energy level E_2 and a higher wavelength λ_2. With $E = h \cdot \nu$,(1) can be transformed into:

$$\frac{1}{h\nu_2} - \frac{1}{h\nu_1} = \frac{1}{m_e c^2}(1 - \cos(\theta)) \qquad (2)$$

With $\lambda = c/\nu$, (2) leads to:

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos(\theta)) \qquad (3)$$

Planck's quantum of action	h = $6.626 \cdot 10^{-34}$ Js
Photon frequency	ν

For the 90° scattering, the difference in wavelength, which only consists of the three universal components, provides the so-called Compton wavelength λ_C for electrons.

$$\lambda_C = \frac{h}{m_e c} = 2.426\,\mathrm{pm}$$

As far as the special cases of the forward and backward scattering by θ = 0° and θ = 180° are concerned, the change in wavelength is $\Delta\lambda = 2\lambda_C$.

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Equipment

Position	Material	Item No.	Quantity
1	XR 4.0 expert unit, 35 kV	09057-99	1
2	XR 4.0 X-ray goniometer	09057-10	1
3	XR4 X-ray Plug-in Cu tube	09057-51	1
4	XR 4.0 X-ray material upgrade set	09165-88	1

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

PHYWE excellence in science

Setup and Procedure

Setup (1/2)

PHYWE excellence in science

- Screw the adapter ring onto the inlet tube of the energy detector and connect the signal and supply cables to the corresponding ports of the detector with the aid of the right-angle plugs.
- Connect the signal and supply cables to the corresponding ports in the experiment chamber of the X-ray unit. In Figure 2, the port for the signal cable is marked in red and the port for the supply cable is marked in green. Connect the external X RED ports of the x-ray unit (see Fig. 3) to the multi-channel analyser (MCA). Connect the signal cable to the "Input" port and the supply cable to the "X-Ray Energy Det." port of the MCA.

Fig. 2: Connectors in the experiment chamber

Fig. 3: Connection of the multi-channel analyser

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Setup (2/2)

- Secure the energy detector in the holder of the swivel arm of the goniometer. Lay the two cables with sufficient length so that the goniometer can be swivelled freely over the entire range.
- Connect the multi-channel analyser and computer with the aid of the USB cable.
- Insert the tube with the 2-mm-aperture.
- Bring the goniometer block and the detector to their respective end positions on the right.

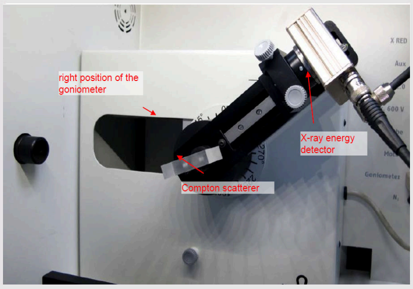


Fig. 4: Goniometer set-up

Procedure (1/5)

- Bring the goniometer block and the detector to their respective end positions on the right.
- Insert the tube with the 1mm-aperture into the exit tube of the X-ray tube.
- With the X-ray unit switched on and the door locked, bring the detector to the 0° position. Then, shift the detector by some tenths degree out of the zero position in order to reduce the total rate.
- Operating data of the tungsten X-ray tube: Select an anode voltage U_A = 25 kV and an anode current I_A = 0.02 mA and confirm these values by pressing the “Enter” button.
- Switch on the X-radiation

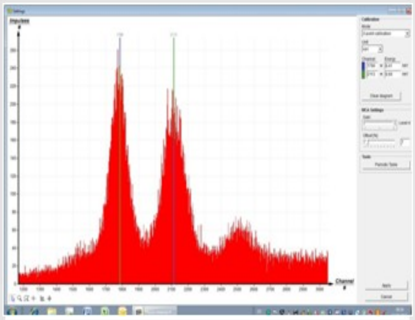


Fig. 5: calibration of the multi-channel analyser

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Procedure (2/5)

PHYWE excellence in science

- In the MEASURE program, select “Multi channel analyser” under “Gauge”. Then, select “Settings and calibration”. After the “Calibrate” button has been clicked, a spectrum can be measured. The counting rate should be < 300 c/s. Energy calibration settings: - 2-point calibration, - Unit = keV, Gain = 2 – Set the offset so that low-energy noise signals will be suppressed (usually a few per cent are sufficient), See Fig 5.
- Measuring time: 5 minutes. Use the timer of the X-ray unit.
- Make the two coloured calibration lines congruent with the line centres of the two characteristic X-ray lines. The corresponding energy values (see e.g. P2544705) $\mathrm{E(L_3M_5/L_3M_4)}$ = 8,41keV and $\mathrm{E(L_2N_4)}$ = 9,69 keV are entered into the corresponding fields, depending on the colour. (Note: Since a separation of the lines $\mathrm{L_3M_4}$ and $\mathrm{L_3M_5}$ Lines is not possible, the mean value of both lines is entered as the energy of the line).
- Name and save the calibration.

Procedure (3/5)

PHYWE excellence in science

Compton scattering

Set the detector to the zero position and select the following operating data: diaphragm tube with d = 1mm, $\mathrm{U_A}$ = 30 kV, $\mathrm{I_A}$ = 0.08 mA.

- Enter the following parameters into the field “Control” in the window “Spectra recording”: - Gain = 2, – Offset = 5%, - X-Data = keV, - Interval width [channels] = 1.
- Start the X-ray tube. The measuring time should be approximately 5 minute so that the intensity of the K_α -peak is approximately 200-300 pulses. Accept the data and save them.
- Place the acrylic glass element (scatterer) of the Compton equipment into the sample holder and set it to a 10° position. Set the detector to 20°.

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Procedure (4/5)

PHYWE excellence in science

- Now, add the tube with the 5 mm aperture and increase the operating data of the X-ray tube to U_A = 35 kV and I_A = 0.3 mA.
- Start the measurement. The measuring time is approximately 10 minutes. The intensity of the K_α-peak should be approximately 200 pulses. Stop the measurement with “Accept data”.
- Leave the acrylic glass scatterer in its position and perform additional measurements. To do so, change the angle of the detector in steps of 10° up to the final value of 160°.

Procedure (5/5)

PHYWE excellence in science

Evaluation of the measurement curves

- In order to determine the line energy, switch from the bar display to the curve display. To do so, click “Display options” and then “Interpolation and straight lines”.
- Extend the relevant line section with the aid of the zoom function
- Then, select the curve section with Open the window “Function fitting Then, select “Scaled normal distribution” and confirm.
- Find the line centroid of the normal distribution with “Peak analysis” or determine it with the function “Survey”

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

PHYWE excellence in science

Evaluation

Task 2

PHYWE excellence in science

Figure 6 shows a part of the X-ray spectrum of molybdenum. For the angle-dependent displacement of energy of the scattered radiation, only the high-intensity L_α-line is to be taken into consideration.

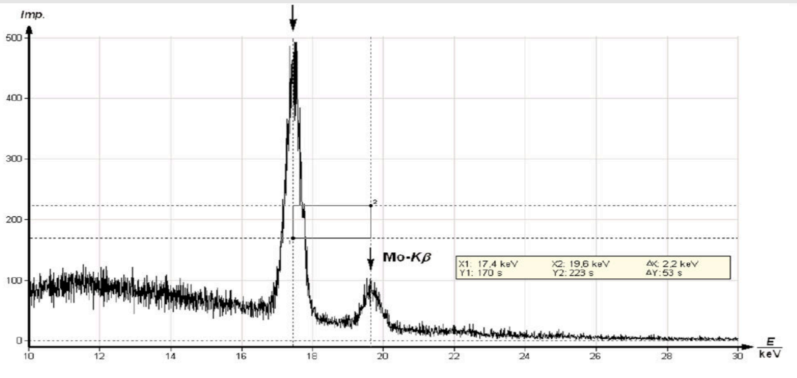


Fig. 6: X-ray spectrum of molybdenum (section)

PHYWE excellence in science

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Task 3

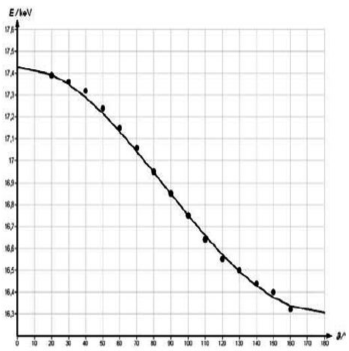


Fig. 7: Energy of the molybdenum K_α-line as a function of the scattering angle.

Column B of table 1 shows the experimental energy values of the line peaks of the W-L_α-line as a function of the scattering angle (column A).

For comparison, the column C shows the energy values that were calculated with $E_1(Mo - K_\alpha) = 17.43\,keV$ based on (1).

Figure 7 shows the content of table 1 in graphical form for clarification.

A	B	C
ϑ / °	E_2 (exp.)/ keV	E_2 (theor.)/ keV
20	17,39	17,394
30	17,36	17,350
40	17,32	17,290
50	17,24	17,218
60	17,15	17,134
70	17,06	17,043
80	16,95	16,947
90	16,85	16,849
100	16,75	16,752
110	16,64	16,659
120	16,55	16,572
130	16,50	16,495
140	16,44	16,429
150	16,40	16,376
160	16,32	16,337

Table 1: Energy E_2 of the scattered photons as a function of the scattering angle θ.

Task 4

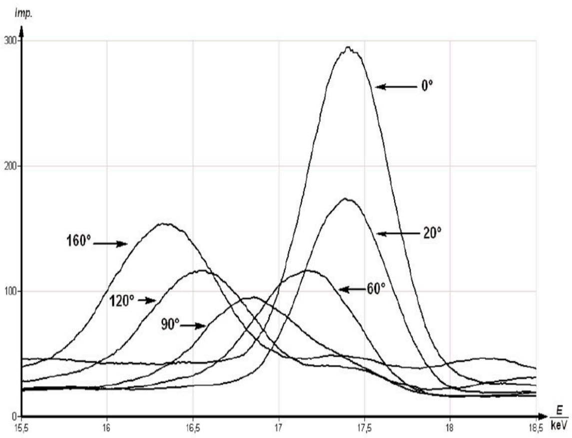


Fig. 8: Molybdenum-K_α-Line of various scattering angles θ.

In order to determine the Compton wavelength λ_C based on the 90° scattering, equation (3) is transformed with $\lambda = hc/E$:

$$\lambda_C = \lambda_2 - \lambda_1 = hc\left(\frac{1}{E_2} - \frac{1}{E_1}\right) \qquad (4)$$

With E_2(90°) = 16.64 keV (see table) and E_1(0°) = 17.43 keV and the equivalence 1 eV = $1.602 \cdot 10^{-19}$ J, one obtains the following Compton wavelength based on the experiment:

$$\lambda_C = 2.49\,pm$$

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

PHYWE excellence in science

Appendix

Conservation of momentum:

PHYWE excellence in science

$\mathrm{p_1 = p_2 + p_3 \;\rightarrow\; p_e^2 = p_1^2 + p_2^2 - 2p_1p_2}$ (5)

The following applies to the angle θ that is formed by the two momentum vectors $\mathrm{p_1}$ and $\mathrm{p_2}$:

$\cos(\theta) = \frac{\mathrm{p_1 p_2}}{\sqrt{|\mathrm{p_1}|^2 |\mathrm{p_2}|^2}}$ (6)

With (6) and the momentum-energy realtions $\mathrm{p_1 = E_1/c}$ and $\mathrm{p_2 = E_2/c}$ (momentum-energy relation based on a combination of $\mathrm{E = h\nu}$, the photon momentum $\mathrm{p = h/\lambda}$ (de Broglie) and $\mathrm{c = \lambda\nu}$), equation (5) leads to:

$\mathrm{p_e^2 = \frac{1}{c^2}(E_1^2 + E_2^2 - 2E_1E_2\cos(\theta))}$ (7)

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Conservation of energy:

PHYWE excellence in science

If one takes the relativistic effects for an electron with the velocity v into consideration, the following results:

$$\mathrm{E}_1 + +\mathrm{m}_0\mathrm{c}^2 = \mathrm{E}_2 + \mathrm{E}_e = \mathrm{E}_2 + \frac{\mathrm{m}_0\mathrm{c}^2}{\sqrt{1-\mathrm{v}^2/\mathrm{c}^2}} \quad (8)$$

With $\mathrm{E} = \mathrm{mc}^2$ and $\mathrm{p}_e = \mathrm{mv}$ it follows:

$$\mathrm{v}^2 = \frac{\mathrm{c}^4\mathrm{p}_e^2}{\mathrm{E}_e^2} \quad (9)$$

If one puts (9) into (8), the following results:

$$\mathrm{p}_e^2 = \frac{1}{\mathrm{c}^2}(\mathrm{E}_1^2 + \mathrm{E}_2^2 + 2\mathrm{m}_0\mathrm{c}^2(\mathrm{E}_1 - \mathrm{E}_2) - 2\mathrm{E}_1\mathrm{E}_2) \quad (10)$$

The combination of (7) and (10) leads to the following for E_2:

$$\mathrm{E}_2 = \frac{\mathrm{E}_1}{1+\frac{\mathrm{E}_1}{\mathrm{m}_e\mathrm{c}^2}(1-\cos(\theta))} \quad (11)$$

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de