

Debye-Scherrer diffractions pattern of powder samples with a diamond structure (according to Bragg-Brentano)



Physics

Modern Physics

Production & use of X-rays



Difficulty level

hard



Group size

2



Preparation time

45+ minutes



Execution time

45+ minutes

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General information

Application

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Setup

Most applications of X rays are based on their ability to pass through matter. Since this ability is dependent on the density of the matter, imaging of the interior of objects and even people becomes possible. This has wide usage in fields such as medicine or security.

Other information (1/2)

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The prior knowledge required for this experiment is found in the Theory section.

Polycrystalline germanium and silicon powder samples, which crystallize in a diamond form, are irradiated with the radiation from a X-ray tube with a copper anode. A swivelling Geiger- Mueller counter tube detects the radiation that is constructively reflected from the various lattice planes of the crystallites. The Debye-Scherrer patterns are automatically recorded. Their evaluation not only allows the Bragg reflexes to be assigned to the individual lattice planes and so also the corresponding Bravais lattice type to be obtained, but also results in the determination of values for their spacings as well as for the lattice constants of the samples and the number of atoms in the unit cell.

Other information (2/2)

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The goal of this experiment is to get to investigate Debye-Scherrer patterns for Bragg-Brentano-geometry.

1. Record the intensity of the Cu X-rays back scattered by a germanium and a silicon powder sample as a function of the scattering angle.
2. Assign the Bragg reflexes to the respective lattice planes.
3. Calculate the lattice plane spacings appropriate to the angular positions of the individual Bragg reflexes. Determine the lattice constants of the samples and their Bravais lattice type.
4. Determine the number of atoms in the unit cell.

Theory (1/4)

When X-rays of wavelength λ strike a mass of lattice planes of a crystal of spacing d at a glancing angle of θ , then the reflected rays will only be subject to constructive interference when Bragg's condition is fulfilled, i.e.:

$$2d \sin(\theta) = n\lambda \quad (n = 1, 2, 3, \dots) \quad (1)$$

Bragg's condition implies that all of the waves scattered at the atom are in phase and so amplify each other, whereas partial waves that are scattered in directions not fulfilling Bragg's conditions are of opposite phase and so extinguish each other. A more realistic way of looking at this must, however take the actual phase relationships of all of the partial waves scattered by the atom in a certain direction into consideration. When there are N atoms in a unit cell, then the total amplitude of the X-rays scattered by the cell is described by the structure factor F , which is calculated by summing up the atomic scattering factors f of the individual N atoms, taking their phases into account.

Theory (2/4)

In general, the following is valid for F :

$$F_{hkl} = \sum_1^N f_n \cdot e^{2\pi i(hu_n + kv_n + lw_n)} \quad (2)$$

where h, k, l = Miller indices of the reflecting lattice planes and u_n, v_n, w_n are the coordinates of the atoms in fractions of the particular edge lengths of the unit cell. As F is in general a complex number, the total scattered intensity is described by $|F_{hkl}|^2$.

The diamond lattice form is distinguished by two fcc lattices which are pushed one within the other, whereby the origin of the second fcc lattice is displaced by $(\frac{1}{4} \frac{1}{4} \frac{1}{4})$ relative to the space diagonals of the first lattice. The unit cell contains 8 atoms, the two origin atoms and 6 additional atoms which lie at the coordinates $(\frac{1}{2} \frac{1}{2} 0), (\frac{1}{2} 0 \frac{1}{2}), (0 \frac{1}{2} \frac{1}{2})$ and $(\frac{3}{4} \frac{3}{4} \frac{1}{4}), (\frac{3}{4} \frac{1}{4} \frac{3}{4}), (\frac{1}{4} \frac{3}{4} \frac{3}{4})$, relative to the origin atoms. The structure factor F for the diamond lattice form can so be calculated from:

Theory (3/4)

$$F = f \left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{4}h + \frac{1}{4}k + \frac{1}{4}l\right)} \cdot 1 + e^{i\pi(h+k)} + e^{i\pi(h+l)} + e^{i\pi(k+l)} \right) \quad (3)$$

The expression in curved brackets is the same as for fcc lattices, i.e. mixed index hkl triplets are not allowed and a value of 4 is obtained for the unmixed indices.

The expression in round brackets is only unequal to zero when $(h + k + l) = 4n$ with $n = 1, 2, 3, \dots$

Theory (4/4)

For the cubic crystal system, the spacing d of the individual lattice planes with the indices (hkl) is obtained from the quadratic form:

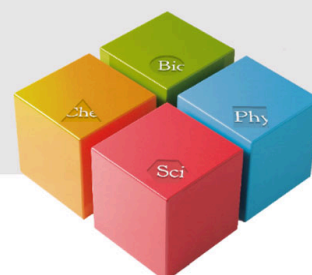
$$\frac{1}{d_{hkl}^2} = \frac{1}{a^2}(h^2 + k^2 + l^2) \quad (a = \text{lattice constant}) \quad (4)$$

From this and equation (1), with $n = 1$, the quadratic Bragg equation is obtained:

$$\sin^2(\theta) = \frac{\lambda^2}{4a^2}(h^2 + k^2 + l^2) \quad (5)$$

Equipment

Position	Material	Item No.	Quantity
1	XR 4.0 expert unit, 35 kV	09057-99	1
2	XR 4.0 X-ray goniometer	09057-10	1
3	XR4 X-ray Plug-in Cu tube	09057-51	1
4	XR 4.0 X-ray structural analysis upgrade set	09145-88	1
5	Germanium, Powder, 99%, 10 g	31768-03	1
6	Silicium, finest powder, 50 g	31155-05	1
7	Vaseline 100 g	30238-10	1

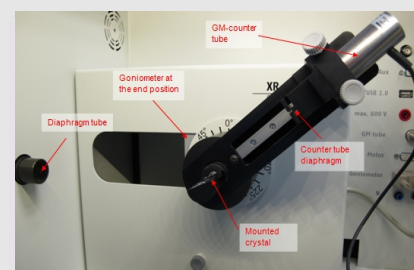
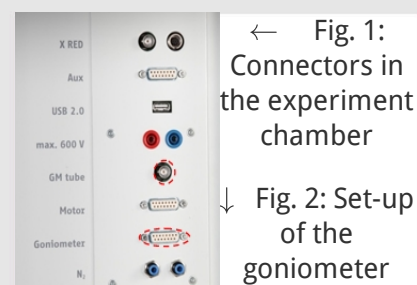


Setup and Procedure

Setup

Connect the goniometer and the Geiger-Müller counter tube to their respective sockets in the experiment chamber (see the red markings in Fig. 1). The goniometer block with the analyser crystal should be located at the end position on the right-hand side. Fasten the Geiger-Müller counter tube with its holder to the back stop of the guide rails. Do not forget to install the diaphragm in front of the counter tube (see Fig. 2). Insert a diaphragm tube with a diameter of 2 mm into the beam outlet of the tube plug-in unit.

For calibration: Make sure, that the correct crystal is entered in the goniometer parameters. Then, select "Menu", "Goniometer", "Autocalibration". The device now determines the optimal positions of the crystal and the goniometer to each other and then the positions of the peaks.



Procedure (1/5)

- Connect the X-ray unit via the USB cable to the USB port of your computer (the correct port of the X-ray unit is marked in Figure 3).
- Start the “measure” program. A virtual X-ray unit will be displayed on the screen.
- You can control the X-ray unit by clicking the various features on and under the virtual X-ray unit. Alternatively, you can also change the parameters at the real X-ray unit. The program will automatically adopt the settings.



Fig. 3: Connection of the computer

Procedure (2/5)

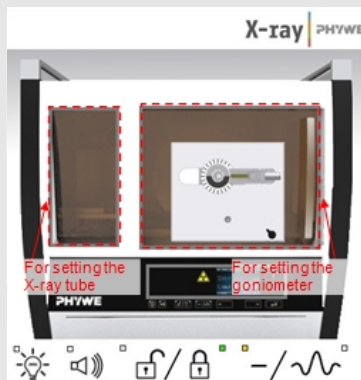


Fig. 4: Part of the user interface of the software

- Click the experiment chamber (see the red marking in Fig. 4) to change the parameters for the experiment.
- If you click the X-ray tube (see the red marking in Figure 4), you can change the voltage and current of the X-ray tube. Select the settings as shown in Figure 5.

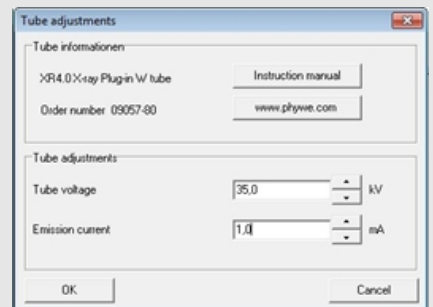
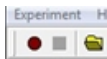


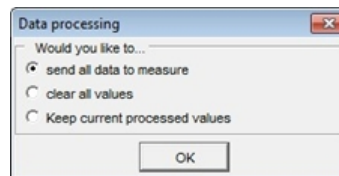
Fig 5: Voltage and current settings

Procedure (3/5)

- Start the measurement by clicking the red circle:



- After the measurement, the following window appears:



- Select the first item and confirm by clicking OK. The measured values will now be transferred directly to the "measure" software.
- At the end of this manual, you will find a brief introduction to the evaluation of the resulting spectra.

Procedure (4/5)

Overview of the settings of the goniometer and X-ray unit:

- 1:2 coupling mode
- angle step width 0.1°
- Scanning range: $10^\circ - 60^\circ$
- Anode voltage $U_A = 35 \text{ kV}$ $U_A=35\text{kV}$; anode current $I_A = 1 \text{ mA}$
- Scanning speed, when only the very intense reflex lines are to be recorded, then scanning can be relatively rapid at 10 s° . For the identification of weaker lines, a scanning speed of at least 40 s° is required for a better signal/noise ratio

Procedure (5/5)

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Sample preparation:

The sample must be so finely powdered that no grains can be felt when a little of it is rubbed between finger and thumb. A relatively high sample concentration can be obtained by mixing the powder with a little vaseline. To do this, transfer a small amount of the sample onto a sheet of paper and use a spatula to knead it to a firm paste. To achieve the highest concentration of material as possible, use very little vaseline (a spatula tip of it). Fill the relatively solid sample paste into the specimen for powder samples and smooth it flush. Use the universal crystal holder to hold the specimen.

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Evaluation

Evaluation of Germanium (1/10)


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Task 1

Fig. 6 shows the Debye-Scherrer spectrum of germanium (Ge).

As no filter is used for the monochromatization of the X-rays, when individual lines are evaluated consideration must be given to the fact that the very intense lines that result from K_α -radiation are accompanied by secondary lines that result from the weaker K_β radiation. These pairs of lines can be identified by means of equation (1). It is namely approximately true with $\lambda(K_\alpha) = 154.18 \text{ pm}$ and $\lambda(K_\beta) = 139.22 \text{ pm}$:

$$\frac{\lambda(K_\alpha)}{\lambda(K_\beta)} = \frac{\sin(\theta_\alpha)}{\sin(\theta_\beta)} \approx 1.1 \quad (6)$$

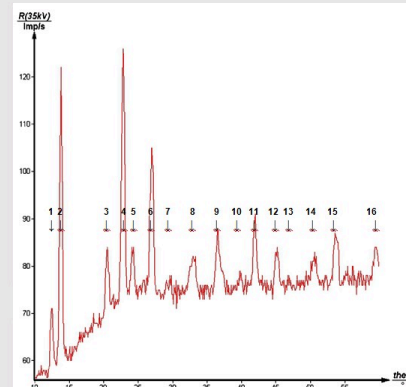


Fig. 6: Bragg Cu – K_α and Cu – K_β lines of Ge.

Evaluation of Germanium (2/10)


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These values correspond to the quotients of the $\sin q$ values (Fig. 6) of the pairs of lines 2-1, 4-3, 6-5, 8-7, 11/10 and 13/12, which shows that the lines 1, 3, 5, 7, 10 and 13 originate from the $\text{Cu}K_\beta$ radiation.

The correctness of this conclusion can be shown by a control measurement (see Fig. 7) using the diaphragm tube with nickel foil to reduce the intensity of the K_β radiation. The reflexes in Fig. 6 that were previously assigned to the K_β lines are no longer to be seen. As the intensity of the K_β -radiation is also somewhat weakened by the Ni foil, the detection of reflexes of weak intensity at larger glancing angles is made difficult when this is used.

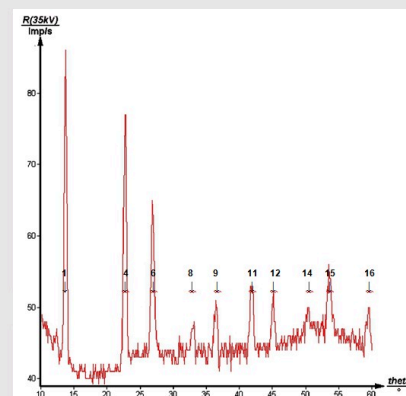


Fig. 7: Bragg-diagram of Ge only with Cu – K_α beam (a nickel filter was used here)

Evaluation of Germanium (3/10)

Task 2

The following method for evaluating the spectrum is given as a representative example for the germanium spectrum. From equation (4), the ratios of the observed $\sin^2(\theta)$ values must be representable by the ratios of the sums of the squares of the three integer numbers (h, k, l).

As shown in column I of Table 1 for the K_{α} - reflexes, calculate the ratios of the \sin^2 values of the individual lines (n) to the \sin^2 values of the first line (here line 2). The numbering in column E relates to the reflex lines indicated in Fig. 6. In column A, all of the possible hkl numbers are listed. Columns B, C and D show the individual ratios of the sums of squares of these numbers.

When an attempt is made to allot the indices 100 or 110 to the first reflexes, then no agreement with the ratios of the $\sin^2(\theta)$ values is found. When the index 111 is assigned to the first line, however, then all of the other lines can be assigned hkl index triplets with a certain accuracy.

Evaluation of Germanium (4/10)

A	B	C	D	E	G	H	I	J	K	L
hkl	$h^2 + k^2 + l^2$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{011}}$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{111}}$	Reflex no.	$\theta / ^\circ$	$\sin(\theta)$	$\sin^2(\theta)$	$\frac{(\sin^2(\theta))_n}{(\sin^2(\theta))_2}$	d / pm	a / pm
100	1									
110	2	1								
111	3	1.5	1	2	14.720	0.2371	0.05625	1.00	325.0256	2.95
200	4	2	1.33							
210	5	2.5	1.67							
211	6	3	2							
220	8	4	2.67	4	22.740	0.3865	0.14942	2.66	199.4356	4.08
221/300	9	4.5	3							
310	10	5	3.33							

Table 1: Evaluation of the K_{α} -Debye-Scherrer lines of Ge.

Evaluation of Germanium (5/10)

A	B	C	D	E	G	H	I	J	K	L	
hkl	$h^2 + k^2 + l^2$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{011}}$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{111}}$	Reflex	no.	$\theta/^\circ$	$\sin(\theta)$	$\sin^2(\theta)$	$\frac{(\sin^2(\theta))_1}{(\sin^2(\theta))_2}$	d / pm	a / pm
311	11	5.5	3.67	6	26.890	0.4522	0.20456	3.64	170.455	65.31	
222	12	6	4								
320	13	6.5	4.33								
321	14	7	4.67								
400	16	8	5.33	8	33.080	0.5458	0.29791	5.30	141.245	64.96	
410/322	17	8.5	5.67								
411/330	18	9	6								
331	19	9.5	6.33	9	36.470	0.5944	0.35331	6.28	129.695	65.32	
420	20	10	6.67								

Table 1: Evaluation of the K_α -Debye-Scherrer lines of Ge.

Evaluation of Germanium (6/10)

A	B	C	D	E	G	H	I	J	K	L	
hkl	$h^2 + k^2 + l^2$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{011}}$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{111}}$	Reflex	no.	$\theta/^\circ$	$\sin(\theta)$	$\sin^2(\theta)$	$\frac{(\sin^2(\theta))_1}{(\sin^2(\theta))_2}$	d / pm	a / pm
421	21	10.5	7								
332	22	11	7.33								
422	24	12	8	11	41.890	0.6677	0.44583	7.93	115.465	65.62	
500/430	25	12.5	8.33								
510/431	26	13	8.67								
511/333	27	13.5	9	12	45.100	0.7083	0.50174	8.92	108.835	65.51	
520/432	29	14.5	9.67								
521	30	15	10								
440	32	16	10.67	14	50.390	0.7704	0.59352	10.55	100.065	66.05	

Table 1: Evaluation of the K_α -Debye-Scherrer lines of Ge.

Evaluation of Germanium (7/10)

A	B	C	D	E	G	H	I	J	K	L
hkl	$h^2 + k^2 + l^2$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{011}}$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{111}}$	Reflex no.	$\theta/^\circ$	$\sin(\theta)$	$\sin^2(\theta)$	$\frac{(\sin^2(\theta))_1}{(\sin^2(\theta))_2}$	d / pm	a / pm
522/441	33	16.5	11							
530/433	34	17	11.33							
531	35	17.5	11.67	15	53.56	0.8044	0.64719	11.51	95.83	566.91
600/442	36	18	12							
610	37	18.5	12.33							
611/532	38	19	12.67							
620	40	20	13.33	16	59.56	0.8621	0.74332	13.21	89.41	565.51

Table 1: Evaluation of the K_α -Debye-Scherrer lines of Ge.

Evaluation of Germanium (8/10)

Task 3

Bravais lattice type:

Only odd indexed hkl triplets occur, or such for which $(h + k + l) = 4n$ is true. According to this, germanium forms a diamond lattice.

Calculation of the lattice plane spacings:

The corresponding lattice plane spacings d calculated using equation (1) are given in column J.

Calculation of the lattice constant :

Column K contains the values for the lattice constant a determined from equation (4). The mean value obtained from the K_α - lines and the K_β lines is:

$$a = (564.56 \pm 1.64) \text{ pm}; \Delta(a)/a < 0.3\% \text{ (literature value } a = 565.75 \text{ pm)}$$

Evaluation of Germanium (9/10)

Task 4

Table 2 gives the results of the evaluation of the $K\beta$ lines 1, 3, 5, 7, 10 and 13 that occur in Fig. 6. On dividing the total mass M of a unit cell by its volume V , the density ρ is given, so that:

$$\rho = \frac{M}{V} = n \cdot m \cdot \frac{1}{a^3} \quad \text{with } m = \frac{m_A}{N} \rightarrow n = \frac{\rho \cdot N \cdot a^3}{m_A} \quad (7)$$

where n = the number of atoms or molecules in the unit cell; m = atomic/molecular mass; $m - A$ = atomic/molecular weight; $N = 6.022 \cdot 10^{23}$ = Avogadro's number. The following are known values for germanium, $\rho = 5.3234 \text{ g/cm}^3$ and $m_A = 72.59 \text{ g}$.

Using these values and $a = 564.56 \text{ pm}$ in equation (7), $n = 7.95 \approx 8$ is obtained, i.e. there are 8 atoms in the unit cell of the diamond lattice form.

Evaluation of Germanium (10/10)

A	B	C	D	E	G	H	I	J	K	L
hkl	$h^2 + k^2 + l^2$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{011}}$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{111}}$	Reflex no.	$\theta / ^\circ$	$\sin(\theta)$	$\sin^2(\theta)$	$\frac{(\sin^2(\theta))_1}{(\sin^2(\theta))_2}$	d / pm	a / pm
111	3	1.5	1	1	12.35	0.2138	0.04575	1.00	325.465	63.72
220	8	4	2.67	3	20.52	0.3505	0.12286	2.69	198.585	61.68
311	11	5.5	3.67	5	24.25	0.4107	0.16869	3.69	169.485	62.11
400	16	8	5.33	7	29.43	0.4913	0.24143	5.28	141.675	66.67
511/333	27	13.5	9	10	39.83	0.6405	0.41026	8.98	108.685	64.71
531	35	17.5	11.67	13	47.11	0.7326	0.53679	11.73	95.01	562.09

Table 2: Evaluation of the $K\beta$ -Debye-Scherrer lines of Ge.

Evaluation of Silicon (1/3)

The Bragg spectrum of silicon, recorded without Ni filter, is shown in Fig. 8. The evaluation of this spectrum carried out according to the above procedure is shown in abbreviated form for the two Cu wavelengths in Tables 3 and 4.

It is again seen here that only uneven hkl index triples occur, or such for which $(h + k + l) = 4n$. According to this, silicon also has a diamond lattice form. Column K contains the values for the lattice constant a determined using equation (4). The mean value from the K_α - and K_β -lines is:

$$a = (543.17 \pm 1.01) \text{ pm}; \Delta(a)/a < 0.2\% \text{ (literature value } a = 543.05 \text{ pm)}$$

Together with the known values for silicon, $\rho = 2.32 \text{ g/cm}^3$ and $m_A = 28.086 \text{ g}$ we again obtain (equation 7): $n = 7.97 \approx 8$

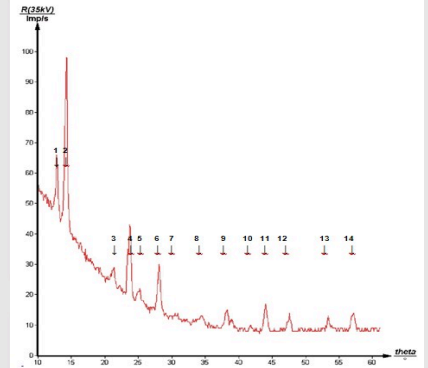


Fig. 8: Bragg Cu – K_α and Cu – K_β lines of Si.

Evaluation of Silicon (2/3)

A	B	C	D	E	G	H	I	J	K	L
hkl	$h^2 + k^2 + l^2$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{011}}$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{111}}$	Reflex	no.	$\theta / ^\circ$	$\sin(\theta)$	$\sin^2(\theta)$	$\frac{(\sin^2(\theta))_n}{(\sin^2(\theta))_2}$	d / pma / pm
111	3	1.5	1	2	14.210	0.2454	0.06026	1.00	314.0454	3.93
220	8	4	2.67	4	23.680	0.4016	0.16131	2.69	191.9454	2.89
311	11	6.5	3.67	6	28.110	0.4711	0.22000	3.65	163.6254	2.67
400	16	8	5.33	8	34.490	0.5662	0.32065	5.32	136.1454	4.56
331	19	9.5	6.33	9	38.260	0.6192	0.38344	6.36	124.4954	2.64
422	24	12	8	11	44.040	0.6951	0.48325	8.00	110.8954	3.25
511/333	27	13.5	9	12	47.600	0.7384	0.54532	9.05	104.3954	2.43
440	32	16	10.67	13	53.420	0.8030	0.64485	10.70	96.00	543.06
531	35	17.5	11.67	14	57.150	0.8400	0.70576	11.71	91.76	542.86

Table 3: Evaluation of the K_α -Debye-Scherrer lines of Si.

Evaluation of Silicon (3/3)

A	B	C	D	E	G	H	I	J	K	L	
hkl	$h^2 + k^2 + l^2$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{011}}$	$\frac{h^2+k^2+l^2}{(h^2+k^2+l^2)_{111}}$	Reflex	no.	$\theta / ^\circ$	$\sin(\theta)$	$\sin^2(\theta)$	$\frac{(\sin^2(\theta))_1}{(\sin^2(\theta))_2}$	d / pm	a / pm
111	3	1.5	1	1	12.83	0.2220	0.04931	1.00	313.485	42.96	
220	8	4	2.67	3	21.34	0.3639	0.13242	2.69	191.295	41.05	
311	11	5.5	3.67	5	25.11	0.4243	0.18001	3.65	164.045	44.06	
400	16	8	5.33	7	30.71	0.5106	0.26081	5.29	136.305	45.20	
511/333	27	13.5	9	10	41.78	0.6662	0.44392	9.00	104.485	42.89	

Table 4: Evaluation of the K_β -Debye-Scherrer lines of Si.